

Bachelor Seminar "Amenable Groups"

SS 2025

In mathematics, an amenable group is a locally compact topological group G carrying a kind of averaging operation on bounded functions that is invariant under translation by group elements. The original definition, in terms of a finitely additive measure (or mean) on subsets of G , was introduced by John von Neumann in 1929 under the German name "messbar" ("measurable" in English) in response to the Banach–Tarski paradox. In 1949 Mahlon M. Day introduced the English translation "amenable", apparently as a pun on "*mean*". Amenable groups arise in ergodic theory, geometric group theory, operator algebras (e.g., in the study of von Neumann algebras), random walks in probability theory, fixed point properties in topology, and statistical mechanics.

•**General information:** This is a 5 ECTS seminar in English for BSc Mathematics (Theoretische Mathematik) and it can also be used as Bachelorseminar (Seminar für Abschlussarbeiten). The purpose of the seminar is to obtain deep insights on amenable groups including examples, non-examples, equivalent characterizations and applications.

•**Lecturer:** Prof. Dr. Kang Li

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Office: 01.376 (by appointment)

•**Time:** Monday 16:15-17:45

•**Room:** Übung 5

•**Form:** Weekly student presentations (2 SWS mandatory attendance). The participants need to choose a favorite topic about amenable groups at the beginning of the semester. By the end of the semester, participants not only submit a written handout about 10 pages but also give a 90 minutes presentation about the chosen topic.

•**Prerequisite:** Basic knowledge of **functional analysis** and **group theory**

•**Topics** could be

1. Means and finitely additive measures [Cellular Automata and Groups, Chapter 4.1-4.2]
2. Amenable groups via means [Cellular Automata and Groups, Chapter 4.3-4.4]
3. Examples of amenable groups [Cellular Automata and Groups, Chapter 4.5-4.6]
4. Visualising amenability: Følner sequences [Cellular Automata and Groups, Chapter 4.7]
5. Non-amenable groups (e.g. free groups) [Cellular Automata and Groups, Chapter 4.8]
6. From means to Følner sequences and back again [Cellular Automata and Groups, Chapter 4.9]
7. The Fixed Point Property [Cellular Automata and Groups, Chapter 4.10+Appendix G]
8. Growth of groups (polynomial, exponential, intermediate) [Cellular Automata and Groups, Chapter 6.4]
9. Introduction to the Banach-Tarski paradox
10. Elementary amenable groups
11. Introduction to Thompson's groups

•**The deadline for the topic selection** is Monday, April 28, 2025 during the seminar 16:15-17:45. **Topics are assigned on a first-come, first-served basis! To request a particular topic in advance, please contact me by email.**

•**Literatures:**

1.Cellular Automata and Groups by Tullio Ceccherini-Silberstein and Michel Coornaert, Springer, 2010.

2.Lecture notes on amenability by Kate Juschenko, see the link:
<https://web.ma.utexas.edu/users/juschenko/notes.html>