**Seminar "Optimization"**

THE PROBABILISTIC METHOD

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The probabilistic method is a proof method that has a wide range of applications in graph theory, number theory and combinatorial geometry. The core idea can be described as follows. To prove the existence of an object with certain properties, one constructs a suitable probability distribution and shows that the probability that a randomly chosen element has the selected property is positive.

A classic example comes from the Ramsey theory. Let $k \geq 3$. How many nodes must a complete graph have, so that any coloring of the edges (let's say with red and blue) contains at least one monochromatic k-clique, that is, a complete subgraph with k nodes and all of its edges are either all blue or all red? Using the probabilistic method it can be shown that the graph must have at least $2^{k/2}$ many nodes. To see this, let n be the number of nodes of the graph, and to obtain a contradiction we assume that $n < 2^{k/2}$. We color all edges independently, with equal probability, red or blue. Now if we consider k nodes, then the probability that all nodes are connected with red edges is $2^{-\binom{k}{2}}$. Thus the probability that any selection of k nodes has a monochromatic coloring is at most $\binom{n}{k} \cdot 2 \cdot 2^{-\binom{k}{2}}$. Now, with our assumption, it holds that $\binom{n}{k} \cdot 2 \cdot 2^{-\binom{k}{2}} < 1$. This means that at least one coloring exists that does not have a monochromatic k-clique. Therefore $n \geq 2^{k/2}$ must apply.
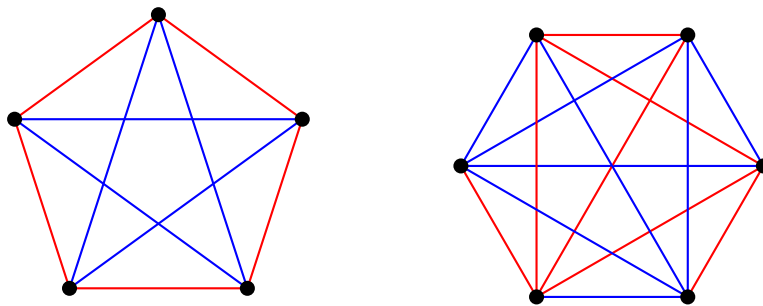
K_5 with a coloring without monochromatic 3-clique. K_6 with a random coloring, with several monochromatic 3-cliques.

The aim of this seminar will be to develop the basic techniques and get to know various applications. The basis for this is the book "The Probabilisitic Method" by Noga Alon and Joel H. Spencer, 2016. A digital version from 2000 is available through the university library. The coursework for the seminar consists of presenting a lecture lasting a maximum of 90 minutes including questions and discussion, a written report of a maximum of 10 pages on your lecture topic and active participation.

If you would like to participate or if you have any questions please contact me by email: **timm.oertel@fau.de**